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K. C. G. Ong, National University of Singapore, Singapore
A. Akbarnezhad*, National University of Singapore, Singapore

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Abstract

The significance of compressive thermal stresses generated during microwave heating on concrete surface delamination is investigated analytically in this paper. Microwave energy acts as an internal heating source that causes concrete to heat up. The amount of microwave energy dissipated in concrete depends on microwave frequency, microwave power and the electromagnetic properties of the concrete. Electromagnetic properties of concrete change as a function of concrete water content, microwave frequency and temperature. The variation in microwave energy dissipation of concrete can be modeled using Lambert’s law which assumes an exponential decay in energy dissipation with distance. The exponent of energy dissipation function depends on the electromagnetic properties of concrete. The decay in microwave energy dissipation gives rise to temperature gradient in the concrete which results in compressive stresses being generated in the heated zone. Another factor that is postulated to affect concrete delamination by microwave heating is the generation of pore pressure within the concrete. However most of the previous studies have overlooked the importance of thermal stresses. In this paper a finite element modeling is used to examine the temperature distribution and thermal stresses generated within the concrete. Results show high compressive stresses being generated especially at high frequencies and in concrete with high water content. The results also reveal that the water content of concrete plays a very important role in the microwave heating process.

Key words: Concrete, Microwaves, Contamination, Heating, Thermal stresses

Introduction

Concrete has been extensively used for the purpose of structural and radiation shielding in nuclear power generation and waste processing plants. As a consequence of such long-term usage, various radionuclides such as strontium, cesium, cobalt, uranium, etc. have gradually diffused into the surface layer of the concrete structure [14]. Typically the contaminated layer is about 1 to 10 mm thick [11]. Demolition of such structures is usually unnecessary as only the radioactive contaminated layer need to be removed and appropriately disposed off, while the bulk of the concrete can be recycled or disposed off as nonradioactive material.
Microwave heating has been recently considered as an efficient replacement for conventional methods in concrete surface decontamination. This method not only increases the efficiency of the decontamination process but also eliminates conventional methods' drawbacks such as dust generation, health hazards, etc.

Microwaves are used in industry primarily as a source of thermal energy. Brief startup periods and rapid uniform heating due to the penetration of microwaves into the product make it an attractive alternative to conventional heating. It is used in the drying of paper, textiles, photographic film and leather. Other uses include oil extraction from tar sands, cross-linking polymers, vulcanization and casting. Perhaps the largest consumer of microwave power is the foods industry where it is used for cooking, thawing, freeze drying, pasteurization, sterilization, etc. Besides, microwave has already been applied in various civil engineering fields such as nondestructive evaluation of materials, effective drying, and accelerated curing of concrete, etc.

While microwaves of low frequency and low power density were shown capable of heating concrete specimens to an almost uniform temperature that can be used in concrete curing, high frequency microwaves have been shown capable of generating a localized field of high stress that can serve as a demolition tool. (Watson 1698; Wace et al. 1989; White et al. 1995)

As a result of dielectric losses, microwaves penetrating a dielectric material act as a volumetrically distributed heat source. Water in concrete is a very strong dipole and is easily heated up by absorbing the microwave energy. As a result the water within the concrete evaporates. When the evaporation rate overtakes the vapor migration rate, pore pressure builds up. Moreover, because of nonuniform heating, especially at high frequencies, creation of local ‘hot spots’ induces high differential stresses. These two phenomena have been postulated to cause delamination of the concrete surface layer.

"Concrete breaking by microwaves" first appeared as the subject of an experimental study in the UK by Watson A. in 1968; thereafter some studies have focused on modeling one dimensional heat phenomenon to justify the experimental observations (Lagos et al. 1995). W. Li et al. 1993 examined the pore pressure and temperature distribution across the concrete sample by considering variable dielectric properties and the microwave frequencies of, 0.896,2.45,10 and 18 GHz. However none of the studies reported in available literature took into account the thermal stresses arising from the high temperature gradient. In a recent published investigation by Bazant et al., the finite volume method was used to determine temperature and pore pressure development as well as thermal stresses in the heated zone. However they did not consider the effects of concrete water content and microwave frequency on the concrete dielectric properties.

Main causes of concrete surface spalling

Available literature suggests two hypotheses dealing with the delamination phenomenon. The first adheres closely to studies on the effects of fire on the pore water pressure in concrete. Modeling of heat and mass transfer seemed to suggest generation of the pore water pressure within the concrete in excess of the concrete tensile strength (Li et al.1993 and Lagos et al. 1995).

The other hypothesis initiated by Bazant et al. (2003), postulates that stresses generated by a differential thermal gradient play a more significant role in the decontamination process. The thermal expansion of the saturated heated zone resisted by the colder surrounding concrete mass, leads to very high compressive stresses parallel to the surface which either crush the concrete, or cause the surface layer to buckle or both. Moreover since the typical porosity of concrete is around 0.1, the actual pore pressures generated are only about a tenth of those predicted by the other researchers and not enough to cause spalling. Bazant et al. also claimed that the effect of pore pressure would in fact be even weaker because of the additional pore space created by the formation of microcracks.

In this paper a finite element model is used to study analytically the significance of thermal stresses in surface delamination of concrete subjected to microwave at different frequencies and power levels. In addition the effects of concrete water content, microwave frequency, microwave power and heating duration are examined.
Electromagnetic properties of materials

To facilitate understanding of the microwave heating phenomenon, the electromagnetic properties of materials used in modeling are briefly described. Every material has a unique set of electromagnetic properties affecting the way in which the material interacts with the electric and the magnetic waves. Concrete is a dielectric (Nonmetallic) material. A dielectric material can be characterized by two independent electromagnetic properties: the complex permittivity $\varepsilon'$ and the complex (magnetic) permeability $\mu'$.

Permittivity

Complex permittivity is defined as:

$$\varepsilon = \varepsilon' - i\varepsilon''$$  \hspace{1cm} (1)

Where $\varepsilon'$, is the real part of complex permittivity and $\varepsilon''$ is the imaginary part of complex permittivity. Dividing this by the permittivity of the free space $\varepsilon_0$, the property becomes dimensionless and relative to the permittivity of free space:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\varepsilon'}{\varepsilon_0} - i\frac{\varepsilon''}{\varepsilon_0}$$  \hspace{1cm} (2)

Or

$$\varepsilon_r = \varepsilon' - i\varepsilon''$$  \hspace{1cm} (3)

Where $\varepsilon_0$, is the permittivity of free space, $\varepsilon_r$ is the relative permittivity, $\varepsilon_r'$ is the dielectric constant and $\varepsilon_r''$ is the loss factor of material. Dielectric constant is a measure of how much energy from an external electric field is stored in a material and the loss factor is a measure of how dissipative or lossy a material is to an external field.

Loss tangent:

The ratio of the energy lost to the energy stored in a material is given as loss tangent:

$$\tan \delta = \frac{\varepsilon''}{\varepsilon'} = \frac{\varepsilon_r''}{\varepsilon_r'}$$

Conductivity

Electromagnetic wave attenuation occurs as a result of conduction current through the lossy dielectric. The equivalent conductivity $\sigma$ can be expressed in terms of the imaginary part of complex permittivity, as follows:

$$\sigma = \varepsilon'' \omega = \left(\varepsilon' \tan \delta\right) \omega = \left(\varepsilon_r' \varepsilon_0 \tan \delta\right) \omega = \left(\varepsilon_r' \varepsilon_0 \tan \delta\right) \left(2\pi f\right)$$  \hspace{1cm} (4)

Where $f$ and $\omega$ are frequency and angular frequency of the electromagnetic wave respectively.

Reflection and transmission of the waves at an interface

The mismatch between the dielectric constants at the interface between two different media causes some of the incident waves to reflect and the rest to be transmitted into the next medium. Mathematical expressions of the reflected wave can be written as:

$$R = \frac{\sqrt{\varepsilon_2 \cos \theta_i} - \sqrt{\varepsilon_1 \cos \theta_2}}{\sqrt{\varepsilon_2 \cos \theta_i} + \sqrt{\varepsilon_1 \cos \theta_2}}$$  \hspace{1cm} (5)

Where $R$ is the reflection coefficient.
Transmissivity

The square of reflection, $|R|^2$, is called reflectivity and denoted as $r$. The transmissivity, $c$, is obtained from:

$$c = 1 - r \quad (6)$$

Heat generation by transverse electromagnetic waves

Generally microwave decontamination of concrete is a near-field microwave problem. As a result, regardless of the selected numerical scheme, the Maxwell equations are generally used to describe the behavior. They govern radiation propagation in a dielectric medium but, owing to their complex formulation, an approximation is used which considers an exponential decay of microwave energy absorption inside the product as prescribed by the Lambert's law. Ayappa et al. (1991) have compared numerical model predictions using Maxwell and Lambert laws to represent power in slabs. They obtained a critical thickness above which the use of Lambert approximation is valid and showed that the two formulations predicted identical power profiles for slabs thicker than 2.7 times the penetration depth which is the case in microwave decontamination especially when high frequencies are used. Similar results were reported by Barringer et al. (1995) that compared predictions by the individual and combined models during heating of gel samples.

Lambert’s law

An expression commonly used for the microwave power term is Lambert’s law. If, $I$ is the transmitted power flux into the medium then the variation $I(z)$ with distance $z$ from the sample surface is

$$I(z) = I_0 e^{-2\beta z} \quad (7)$$

Where, $I_0$ is the initial power of microwave and $\beta$ is the attenuation factor for a given material and microwave frequency that can be computed from

$$\beta = \omega \sqrt{\pi \varepsilon} \left\{ \frac{1}{2} \left[ 1 + \left( \frac{\sigma}{\omega \varepsilon} \right)^2 \right] - 1 \right\}^{\frac{1}{2}} \quad (8)$$

The power absorbed per unit volume at distance $z$ is

$$PL(z) = -\frac{\partial I(z)}{\partial z} = 2\beta I_0 e^{-2\beta z} \quad (9)$$

The use of Lambert’s law requires an estimate of the transmitted power intensity $I$, which is obtained from calorimetric measurements (Taoukis et al., 1987; Ohlsson and Bengtsson, 1971) or used as an adjustable parameter to match experimental temperature profiles with model predictions (Nykvist and Decareau, 1976). Thus $I$, measured by the above methods represents the intensity of transmitted radiation. The accuracy of the estimate depends on the method used. Alternately if $I$, is the incident power flux then Lambert’s law must be modified to account for the decrease in power, due to reflection at the surface of the sample. In this paper a transmission coefficient, $c$, is applied to modify Lambert’s law to account for the loss in microwave energy due to reflection.

$$PL(z) = -\frac{\partial I(z)}{\partial z} \times c = 2\beta I_0 e^{-2\beta z} \times c \quad (10)$$
Model description

A 0.6m thick concrete wall and a 0.6×0.6m square microwave applicator are considered. Therefore a 0.6×0.6×0.6m cube exposed to microwave from one surface is modeled. The mechanical and thermal properties of concrete used in modeling are shown in Table 1.

Heat Transfer

The governing equation of a typical microwave heating problem is the heat transfer equation, where the microwave energy dissipation appears as the internal heat source.\[ \rho c \frac{\partial T}{\partial t} = -\nabla \cdot (K \nabla T) + PL(z) \] (11)

Here, \( \rho \) = mass density of concrete, \( t \) = time, \( \nabla \) = gradient operator, \( c \) = specific heat of concrete, \( K \) = heat conductivity, \( T \) = temperature and \( PL(z) \) = energy from microwave heating source.

Thermal boundary conditions

For the microwave-exposed surface and the distal surface of the concrete wall, convection with ambient is imposed as the boundary condition. Moreover, for the other four surfaces of the heated zone, appropriate convection and conduction heat transfer with unheated parts of concrete wall are considered.

Structural boundary conditions

As a part of a big wall, the heated zone displacement is restricted by the unheated parts of the wall and therefore, simply supported boundary condition may be considered:

\[ U_x = U_y = U_z = 0 \] (12)

However, in this paper both free and simply supported boundary conditions are examined and the results are compared.

Input data

The amount of the energy dissipated in concrete depends completely on its electromagnetic properties. Electromagnetic properties of concrete are a function of factors such as concrete ingredients and mix proportions, water content, microwave frequency, temperature, etc. However, despite the significant effect of concrete water content and the microwave frequency on heating process they seem to have been overlooked in previous studies. W. Li et al. took the frequency and temperature dependence of dielectric properties into the account. However they overlooked the effect of concrete water content. Bazant et al. simply assumed that the volume fraction of water content of concrete in nuclear facilities is about 7% and seemingly ignored the dependence of concrete electromagnetic properties on the microwave frequency. In this study the available electromagnetic permittivity and permeability data from Ref. [13] are used. These data as well as calculated data for attenuation factors and transmission coefficients used in the calculations are shown Fig 1.

Water content of concrete

Water plays an inevitable role in concrete’s electromagnetic properties. The energy absorption capacity of concrete changes significantly with any change in its water content. Therefore, prior to any investigation in microwave decontamination of concrete, the water content of concrete and its corresponding electromagnetic properties should be measured. In this paper, in order to study the significance of this factor, four different types of concrete are modeled: 1) wet specimen standing water on its surface 2) saturated, surface dry concrete 3) air-dried concrete exposed to ambient room temperature and humidity 4) oven-dried concrete with zero moisture content by weight [13]. The corresponding electromagnetic properties for each type of concrete are shown in Fig. 1.
Table 1. Mechanical and thermal properties of concrete

<table>
<thead>
<tr>
<th>Property</th>
<th>Assumption</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement type</td>
<td>Portland cement type I</td>
<td></td>
</tr>
<tr>
<td>Water/cement/sand/coarse agg. Mix ratio of concrete</td>
<td>1:2.22:5.61:7.12</td>
<td>By weight</td>
</tr>
<tr>
<td>Maximum aggregate size</td>
<td>3.81</td>
<td>cm</td>
</tr>
<tr>
<td>Density of concrete</td>
<td>2300</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Heat capacity</td>
<td>1000</td>
<td>J/kg·°C</td>
</tr>
<tr>
<td>Heat conductivity</td>
<td>3.67</td>
<td>J/m²·s·°C</td>
</tr>
<tr>
<td>Thermal expansion coefficient</td>
<td>12.0 × 10⁻⁶</td>
<td></td>
</tr>
<tr>
<td>Heat transfer coefficient of heat flux</td>
<td>10.0</td>
<td>J/m²·s·°C</td>
</tr>
<tr>
<td>Frequency range</td>
<td>1-20</td>
<td>GHz</td>
</tr>
<tr>
<td>Initial power range</td>
<td>0.5-3</td>
<td>MW</td>
</tr>
<tr>
<td>Elasticity</td>
<td>48.5</td>
<td>GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>Surface emissivity</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Initial Temperature</td>
<td>25</td>
<td>°C</td>
</tr>
</tbody>
</table>

Fig. 1. (a) Relative dielectric constant of concrete vs. microwave frequency; (b) Loss factor of concrete vs. microwave frequency; (c) Attenuation factor of concrete vs. microwave frequency; (d) Transmission factor of concrete vs. microwave frequency.
Microwave frequency range

Since, it is impossible to plot the results for all frequencies and in order to make it possible to compare the results with other available results, in most of the parts of this study, the common frequencies of 3, 10 and 18 GHz are considered. These three frequencies can fairly represent the characteristics of typical low, intermediate and high frequencies.

Mesh grid generation

As can be seen in Figs. 4 and 5, the microwave power dissipation pattern varies from almost uniform for dry concrete subjected to low frequencies to a very sharp curve for wet concrete subjected to high frequencies. In order to achieve an acceptable accuracy within a reasonable computation time, appropriate mesh densities should be considered for each case. Since at high microwave frequencies, most of the microwave energy will be dissipated within a small layer near the surface of the concrete, a denser mesh is used near the surface.

Microwave power dissipation

The energy dissipation function for each type of concrete can be approximated using calculated attenuation factors (Fig. 1) and Equation (12). For the sake of comparison, relative energy dissipation factor \( \frac{P_L(z)}{I_0} \) for three commonly used microwave frequencies and two different water content conditions are shown in Figs. 4 and 5. As can be seen, unlike low frequencies, at high frequencies, the dimensionless energy dissipation factor in concrete with a specific water content, decreases rapidly with distance from the microwave-exposed surface. Moreover, by comparing saturated and air-dried concrete it is deduced that for a specific microwave frequency much more energy is dissipated in a concrete with higher water content.
Results and discussion

Temperature distribution

As a result of microwave energy dissipation, the microwave-exposed area of the concrete is heated up rapidly. According to Equation (10), the microwave energy dissipation in any material depends on its attenuation and transmission factor values. Moreover, these two factors vary with microwave frequency and the water content of the concrete. As a result, before any analytical or experimental investigation of a typical microwave decontamination case, microwave frequency and the concrete’s water content should be determined. Temperature distribution and its variation with microwave frequency and the concrete’s water content are plotted in Figs. 7 to 9. In order to make the plots more legible, only the results of analysis at 10 and 18GHz are plotted for Z=0 to 0.2m. As was expected, the temperature distribution patterns for different frequencies and different water contents of concrete are similar to those of microwave power dissipation at the same conditions.

As can be seen in Figs. 7 to 9, for a specific microwave power (2.4MW), at higher frequencies and higher water contents, microwave penetration is reduced as more energy is dissipated near the concrete surface. This results in higher temperature and temperature gradients within a thin surface layer of the concrete sample. However, in concretes with low water contents at all frequencies and also in all types of concrete at low frequencies, the temperature gradient decreases appreciably. This is borne by the fact that low frequencies have already been used to generate a uniform temperature distribution in concrete for curing purposes.

Thermal stresses

Temperature gradient in the heated zone of concrete results in nonuniform expansion in this area and leads to very high compressive stresses. As mentioned earlier, since the heated zone is considered as a part of a concrete wall, the boundaries of the heated zone should be assumed to be fixed. However, the results of finite element modeling show that because of high temperature gradient especially at high frequencies, even in the case of concrete sample with free structural boundary condition, very high compressive stresses are generated.

The results of the thermal stress analysis for different microwave frequencies and concrete’s water contents, for both supported and free boundary conditions are shown in Figs. 10 to 15. For a concrete with definite nominal strength, the depth of spalling for a specific microwave power, microwave frequency and concrete water content, can be simply estimated by drawing a horizontal line at the given strength.

Fig. 6. Sketch of microwave decontamination system
Effect of microwave frequency

Generally, for a given microwave power and heating period, higher microwave frequencies result in higher stresses within a thinner surface layer of concrete. The effect of microwave frequency on the maximum compressive stress for wet, saturated and air-dried concrete are shown in Fig. 16. As can be seen, especially for wet concrete and frequencies between 5 to 15 GHz, an increase in microwave frequencies can lead to significantly higher compressive stresses. However the sensitivity to microwave frequency decreases with a decrease in concrete’s water content.

Effect of microwave power and microwave heating period

Figs.17 and 18, reveal that for a given microwave frequency and concrete type, the maximum compressive stress in the concrete increases linearly with microwave initial power.

In addition, as can be seen in Figs 19 and 20, for all types of concrete and microwave frequencies, the maximum temperature and compressive stress reached in concrete vary almost linearly proportional with the heating time.

Conclusions

1-The results confirm the significance of thermal stresses in concrete surface delamination as the magnitude of the stresses generated can be relatively high.
2-Experimental investigations are necessary to confirm this and to study the concomitant effects of pore pressure.
3-Results reveal high sensitivity of microwave heating process to the water content of concrete. Therefore prior to any analytical or experimental investigation, the water content of concrete and its corresponding electromagnetic properties should be measured.
4-The spalling depth of the concrete surface layer and the necessary spalling time are inversely proportional to microwave frequency.
5-Temperature and stress generated in concrete seemed to vary proportionally with the microwave initial power and heating duration.
Fig. 7. Temperature distribution for concrete at the frequency of 3GHz and microwave power of 2.4 MW after 20 seconds.

Fig. 8. Temperature distribution for concrete at the frequency of 10 GHz and microwave power of 2.4 MW after 5 seconds.

Fig. 9. Temperature distribution for concrete at the frequency of 18GHz and microwave power of 2.4 MW after 1 second.

Fig. 10. Compressive stress in concrete at the frequency of 3GHz and microwave power of 2.4 MW after 20 seconds under simply supported boundary conditions.

Fig. 11. Compressive stress in concrete at the frequency of 10 GHz and microwave power of 2.4 MW after 5 seconds under simply supported boundary conditions.

Fig. 12. Compressive stress in concrete at the frequency of 18GHz and microwave power of 2.4 MW after 1 second under simply supported boundary conditions.
Fig. 13. Compressive stress in concrete at the frequency of 3GHz and microwave power of 2.4 MW after 20 seconds under free boundary conditions.

Fig. 14. Compressive stress in concrete at the frequency of 10GHz and microwave power of 2.4 MW after 5 seconds under free boundary conditions.

Fig. 15. Compressive stress in concrete at the frequency of 18GHz and microwave power of 2.4 MW after 1 second under free boundary conditions.

Fig. 16. Maximum compressive stress vs. Microwave frequency at microwave power of 2.4 MW and after 20, 5 and 1 s for wet, saturated and air-dried concrete respectively.

Fig. 17. Maximum compressive stress vs. initial microwave power at the frequency of 3 GHz after 2 seconds.

Fig. 18. Maximum compressive stress vs. initial microwave power at the frequency of 10 GHz after 2 seconds.
Appendix

Conduction and convection heat transfer formulation in finite element

The first law of thermodynamics states that energy is conserved. Specializing this to a differential control volume:

\[
\rho c \left( \frac{\partial T}{\partial t} + \{V\}^T \{L\}^T \right) + \{L\}^T \{q\} = \dot{q}
\]

(A-1)

Where:
\( \rho \) = density
\( c \) = specific heat
\( T \) = temperature
\( t \) = time

\[ \{L\} = \begin{bmatrix} \frac{\partial}{\partial X} \\ \frac{\partial}{\partial Y} \\ \frac{\partial}{\partial Z} \end{bmatrix} \] = Vector operator

\[ \{V\} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} \] = Velocity vector of heat and mass transport

\[ \{q\} \] = Heat flux vector

\( \dot{q} \) = heat generation per unit volume (microwave energy)

Next, Fourier’s law is used to relate the heat flux vector to the thermal gradients:
\{q\} = -[D]\{L\}T \quad (A-2)

Where:

\[
[D] = \begin{bmatrix}
K_{xx} & 0 & 0 \\
0 & K_{yy} & 0 \\
0 & 0 & K_{zz}
\end{bmatrix} = \text{conductivity matrix}
\]

\(K_{xx}, K_{yy}, K_{zz}\) = Conductivity in the element x, y, and z directions respectively.

Combining Equation (A-1) and (A-2):

\[
\rho c \left( \frac{\partial T}{\partial t} + \{V\}^T \{L\}^T \right) = \{L\}^T \left( [D]\{L\}T \right) + \bar{q} \quad (A-3)
\]

Expanding equation (A-3) to a more familiar form:

\[
\rho c \left( \frac{\partial T}{\partial t} + V_x \frac{\partial T}{\partial X} + V_y \frac{\partial T}{\partial Y} + V_z \frac{\partial T}{\partial Z} \right) = q + \frac{\partial}{\partial X} \left( K_x \frac{\partial T}{\partial X} \right) + \frac{\partial}{\partial Y} \left( K_y \frac{\partial T}{\partial Y} \right) + \frac{\partial}{\partial Z} \left( K_z \frac{\partial T}{\partial Z} \right) \quad (A-4)
\]

It may be assumed that all effects are in the global Cartesian system. Three types of boundary conditions are considered. It is presumed that these cover the entire element.

(a) Specified temperatures acting over surface S1

\[T = T^* \quad (A-5)\]

Where \(T^*\) is the specified temperature

(b) Specified heat flows acting over surface S2

\[\{q\}^T \{\eta\} = -q^* \quad (A-6)\]

Where:

\{\eta\} = \text{unit outward normal vector}

\(q^*\) = specified heat flow

(c) Specified convection surfaces acting over surface S3

\[\{q\}^T \{\eta\} = h_f \left( T_s - T_B \right) \quad (A-7)\]

Where:

\(h_f\) = film coefficient evaluated at \((T_B + T_s)/2\).

\(T_B = \text{bulk temperature of the adjacent fluid}\)

\(T_s = \text{temperature at the surface of the model}\)

Note that positive specified heat flow is into the boundary (i.e., in the direction opposing \{\eta\}), which accounts for the negative signs in Equation (A-6) and Equation (A-7).

Combining Equation (A-2) with Equation (A-6) and Equation (A-7)

\[\{\eta\}^T [D]\{L\}T = q^* \quad (A-8)\]

\[\{\eta\}^T [D]\{L\}T = h_f \left( T_s - T_B \right) \quad (A-9)\]
Premultiplying Equation (A-3) by a virtual change in temperature, integrating over the volume of the element, and combining with Equation (A-8) and Equation (A-9) with some manipulation yields:

\[
\int_{V_{ol}} \left( \rho c \partial T / \partial t + \{V\}^T \{L\}^T \{L\} \right) d(vol) = \int_{S_2} \delta T q^* d(s_2) + \int_{S_3} \delta T h_f (T_B - T) d(S_3) + \int_{V_{ol}} \delta T \bar{q} d(vol) \quad (A-10)
\]

Where:

- \( V_{ol} \) = volume of the element
- \( \delta T \) = an allowable virtual temperature

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