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EFFECTS OF COLUMN SHORTENING IN TALL R.C. BUILDINGS

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Abstract

In tall and very tall reinforced concrete buildings, long term column shortening due to creep and shrinkage affects the service life behavior of structural elements. Creep-induced column shortening depends on the axial stress level in the column cross sections and on its evolution in time. For this reason, the construction method and phases must be taken into account as a key factor in defining the evolution of the stresses. If approached in its comprehensive theoretical formulation, the problem of column shortening in tall buildings exhibits high computational and analytical complexity, because the different construction phases must be taken into account, together with the loading history and the rheological non-homogeneity of the different concrete elements at the times when loading is applied. Anyway, to approach the general problem more easily, a few assumptions can be made allowing to derive, by means of simplified calculations, the upper and lower boundaries of the exact general analytical solution. In the present paper, at first, the problem of long time column shortening will be discussed, then approximate solutions will be derived and applied with reference to the case study of Palazzo Lombardia, at present the tallest building in Italy.

Keywords: Tall buildings, concrete, creep, shrinkage, ageing behavior, axial load

1. Introduction

The long term shortening of columns in tall buildings with reinforced concrete structural elements is an important aspect in the design of such buildings. In fact, because of column shortening, the slab systems undergo differential displacements of their supports, which induce stress redistributions that must be carefully investigated. Moreover, non-structural elements, in particular glass facades, might undergo deformations that are not compliant with the tolerances required in design, if differential shortening is neglected. The problem of column shortening, due to creep and shrinkage of concrete, was studied in /1/, /2/, /3/, during the seventies, when the buildings started to become increasingly high. Nonetheless, most of the tall buildings at that time had vertical structural elements made of steel, so that differential shortening was only related to the elastic behavior of columns and the construction methods, while the analysis of long term effects were limited to the study of the differential shortening between the steel columns and the concrete stairway core. Starting from the new millennium, the development of high strength concrete allowed tall buildings to be built entirely of concrete structural elements, so that the problems related to differential shortening have become more and more important to the present day, /4/. Even if state-of-the-art structural analysis software is
today available to the designer to evaluate long term effects of column shortening, keeping track of the different construction phases, nonetheless the availability of simple and reliable closed-form solutions, able to predict the boundaries of long term behavior, is very important in the design and pre-dimensioning phase. In order to contribute to the solution of this interesting problem, in the present paper the problem is approached in its general form, providing the analytical solution in the framework of the viscoelastic theory, with the assumption of models for the behavior of materials that are able to maximize the effects of column shortening, in order to achieve upper bounds for the general theoretical solution. In particular, some fundamental rules will be derived, necessary to define the upper and lower bounds for the stress distribution in the slabs without recurring to complex formulations. In this way, it is possible to use standard structural analysis software, without taking into account the construction phases, to run equivalent elastic analyses only to define the range of variation of long term stress distribution, which is very useful in the design phase. The formulations that will be presented in the following will then be applied for the analysis of the stress and deformation patterns of the Palazzo Lombardia, recently built in Milano, represented in Fig. 1 and currently the tallest building in Italy, for which the First Author served as the structural designer and general design coordinator. The analyses will be presented and some theoretical aspects of the proposed formulation will be discussed in detail with regards to the case study, deriving some principles of general validity for a correct approach to the conceptual design of tall buildings.

![Fig. 1 – Palazzo Lombardia Complex: view of the Tower Building](image)

2. **Analysis of long term deformations in reinforced concrete columns**

The permanent stress distribution in reinforced concrete columns causes, as a consequence of the creep of the material, an increase in the axial deformation of the cross sections which, due to the remarkable height of the column itself, gives way to important increases in the vertical displacements of the column. Because of the different stress distribution in the various columns and in particular the difference in stresses between the columns and the core, the increase in the vertical displacements of columns become differential displacements of the supports of the floor slabs, which end up undergoing increased stresses with respect to those predicted when the supports, i.e. the columns and the core, are assumed to be fixed, as generally is done in standard structural analysis of slabs. In order to achieve a suitably reliable evaluation of the increase of vertical displacements of columns due to creep and shrinkage of concrete, in the following an approximate solution is formulated, which allows a safe-side estimation by means of simple calculations. This approach allows the upper
bounds for the differential displacements of columns to be defined and the range of variation of the stresses in the slab. In order to simplify the calculations, the following assumptions are made:

- the axial stress in the columns is not affected by the variations of the stress distribution in the slabs;
- there is no interaction between the variations in the stress distribution in the various slabs;
- the creep behavior of concrete is described by the ageing, rate of creep model;
- for the structural and sectional analyses of concrete columns and slabs the algebraic formulation of the constitutive law of concrete is used, as defined in CEB/FIP Model Code 90, /5/ and in EuroCode 2, /6/;
- the analyses are carried out taking into account the construction phases along the height of the building.

The first two assumptions are due to the generally small flexural stiffness of the slabs, which, even under significant values of shear at their ends, only gives way to negligible variation of the axial stresses in columns. The third assumption approaches the problem from the safe side, since a larger material inhomogeneity is assumed when using the rate of creep model, so that the variations in the stresses in the slabs due to the displacements of columns are overestimated. The last assumption is necessary in order to correctly describe the stress pattern that progressively appears in the columns, whose age varies while the loading process progresses. Moreover, the description of the loading process on a phase by phase basis allows the actual vertical displacements of columns to be taken into account, while of course when each slab is cast during construction, said displacements are annulled. On the basis of the above mentioned assumptions, and assuming also, as a simplification, that, given the high number of floors, the variation of the axial stress in columns varies uniformly along the height, with a reference to Fig.2, taking into account that, when the slab at the height z is cast, the deformations caused by all the pre-applied loads are annulled, for the axial force in the column and for the displacement induced in it by the subsequently applied loads, assuming a linear elastic behaviour of concrete, one can write:

\[ N(z)=q(l-z)=ql(1-\xi) \quad (1) \]

\[ u_{zz}(z)=q(l-z)z/(EA)=(ql^2/(EA))x(1-\xi)\xi ; \quad \xi=z/l \quad (2) \]

The displacement \( u_{zz}(\xi) \), represented by the black line in Fig.3, consists in a parabolic curve with its maximum at half of the height of the column, having the value \( u_{1\text{max}}=0.25ql^2/(EA) \). If the displacement under the total load is calculated, as if the global vertical load is applied without taking into account the construction phases, the displacement at the height z becomes:

\[ u_{zz}(q^2/E\xi)(1-\xi/2) \quad (3) \]

represented by the red line in Fig.3, which consists in a parabolic curve with its maximum at the top of the column, having the value \( u_{2\text{max}}=0.5ql^2/(EA) \). It can thus be observed that keeping into account the construction phases reduces the maximum vertical displacement to a half of the maximum value computed by means of a conventional analysis.

In the linear viscoelastic filed, assuming the rate of creep model, whose constitutive law, /7/, can be written:

\[ \frac{d\varepsilon}{dt}(t,t)=\frac{1}{E}(\frac{d\sigma}{dt}(t,t)) + \sigma/E \quad (4) \]

being

\[ \Phi(t,t)=\Phi_\infty (1-e^{-\beta(t-t_0)/}) \quad (5) \]

the basic creep coefficient, referred to concrete at a height \( z=l \). In Eq. (5), \( \Phi_\infty \) represents the maximum value, \( \beta \) is the reverse of the delay time \( \tau^* \), assumed equal to one year, and \( t_0 \) is the time when it is first possible to load the structure.
Defining as $\Phi_z(t, t_z)$ the creep coefficient of the material at the height $z$, the rate of creep model allows the following relation to be written:

$$\Phi_z(t, t_z) = \Phi_\infty (1 - e^{-\beta(t - t_z)})$$  \hspace{1cm} (6)

so that, comparing Eq. (5) and Eq. (6), it can be derived

$$d\Phi_z(t, t_z)/dt = e^{-\beta(t - t_z)} d\Phi(t, t_0)/dt$$ \hspace{1cm} (7)

where $t_0 - t_z$ represents the difference in age between the concrete at height $z$ and that at height $z=l$.

Defining as $T$ the time of construction of the building, and assuming that the construction progresses at constant speed $v$, the following relations hold true

$l = vT$ \hspace{1cm} $z = vt_z$

from which

$t_z = zT/l = T \xi$

With reference to the height $z^*$, calling $t_0$ the time when concrete at a height $z > z^*$ can be at first loaded, we can write

$$d\Phi_z(t, t_z)/dt = e^{-\beta(T - \xi)} d\Phi(t, t_0)/dt$$ \hspace{1cm} (8)

and, integrating Eq. (7), it can be derived

$$\Phi_z(t, t_z) = e^{\beta(t_0 - t_z)} \Phi(t, t_0)$$ \hspace{1cm} (9)

Eq. (9) is represented in Fig. 4 for various values of the age difference ($t_0 - t_z$). It can be noted that, when the age difference increases, the creep induced deformation decreases, because the elder part of the structure is the one to which the basic creep coefficient is referred.
Fig.4 – Rate of creep model, variation of the creep coefficient with the material age

By applying Eq. (8), the following relation can be derived for the creep induced displacement of the cross section at height \( z \), for the loads applied in the interval \( 0 \leq z^* \leq z \):

\[
u_{z1} = \left( \frac{q^2}{EA} \right) \Phi(t, t_0) \int_0^\xi e^{-\beta T(\xi-\xi^*)} d\xi^* = \left( \frac{q^2}{EA} \right) \Phi(t, t_0) \left( 1-e^{\beta T(1-\xi)} \right) / (\beta T)^2
\]

In the same way, for the actions applied above the height \( z \), varying in time according to the relation

\[
q(t) = q(t-t_z)/T \quad t_z \leq t \leq T
\]

or also, as a function of the abscissa of the progress of the construction

\[
q(\xi, \xi^*) = q(\xi-\xi^*) \quad \xi \leq \xi^* \leq 1
\]

integrating Eq. (3), with the loading law (11),(12) for the vertical displacement at the height \( z \), one can obtain

\[
u_{z2}(\xi) = \left( \frac{q^2}{EA} \right) \Phi(t, t_0) \left( 1-\xi \right) (1+\Phi(t, t_0) g_2(\xi))
\]

Introducing the functions

\[
g_1(\xi) = 2 \left( 1-e^{\beta T(1-\xi)} \right) / (\xi^2 (\beta T)^2)
\]

\[
g_2(\xi) = \left( 1-e^{\beta T(1-\xi)} + e^{\beta T} \right) / (\xi (1-\xi) (\beta T)^2)
\]

the displacements \( u_{z1}, u_{z2} \) become

\[
u_{z1}(\xi, t) = u_0(\xi^2/2) \Phi(t, t_0) g_1(\xi)
\]

\[
u_{z2}(\xi, t) = u_0(1-\xi)(1+\Phi(t, t_0) g_2(\xi))
\]

being

\[
u_0 = \left( \frac{q^2}{EA} \right)
\]

Remembering that at the initial time \( t=t_0 \), when, by virtue of Eq. (3) it is \( \Phi=0 \) and the structural behaviour is elastic, the related displacements are given by the expressions:

\[
u_{z1}(\xi) = u_0(\xi^2/2)
\]

\[
u_{z2}(\xi) = u_0(1-\xi)\xi
\]

for the total viscoelastic vertical displacement \( u_{z3} \) at a height \( z \), due to the pre-applied loads and to those subsequently applied, by virtue of the principle of superposition we can write

\[
u_{z3}(\xi, t) = u_{z1}(\xi) \Phi(t, t_0) g_1(\xi) + u_{z2}(\xi)(1+\Phi(t, t_0) g_2(\xi))
\]
The above derived relationships allow the vertical displacements of columns under permanent actions to be computed and their time history to be followed. As for the evaluation of the variation of the flexural redundant reactions in the slabs under permanent actions, due to the column shortening, this can be obtained by superposing the reactions evaluated when the columns are considered infinitely rigid to the ones caused by the displacements of the columns. In order to evaluate the latter reactions, referring only to the pre-applied actions at height \( z \), the slab can be considered as a delayed restraint on the column. The system of compatibility Volterra integral equations for the slab beams can so be written

\[
\int_0^t F_e dX_1(\xi,t') J(t,t') = u_{ze1}(\xi) g_1(\xi) \Phi(t,t_0)
\]  

being respectively \( F_e \), \( X_1 \), \( u_{ze1} \) the flexibility matrix of the beam, the vector of the statically indeterminate reactions and the vector of the displacements of the columns induced by creep.

The solution of Eq. (22), remembering the basic theorems of linear viscoelasticity, /8/, and defining as \( R(t,t_0) \) the relaxation function, can be written as

\[
X_1(\xi,t) = X_{e \text{ post}}(\xi) g_1(\xi)(1-R(t,t_0)/E)
\]

being

\[
X_{e \text{ post}}(\xi) = F_e^{-1} u_{ze1}(\xi)
\]

the vector of the statically indeterminate reactions computed in the elastic field, assuming that the slab is pre-existent with respect the axial loads applied below the abscissa \( z \). For the part of the loads applied after the slab and the columns are connected, for which the slab acts as a pre-existing restraint, the system of Volterra integral compatibility equations becomes

\[
\int_0^t F_e dX_2(\xi,t') J(t,t') = u_{ze2}(\xi)(1+\Phi(t,t_0) g_2(\xi))
\]

Remembering that \( J(t,t_0) = (1+\Phi(t,t_0))/E \) and introducing the elastic solution for a slab pre-existing to the applied actions, given by the expression:

\[
X_{e \text{ pres}}(\xi) = F_e^{-1} u_{ze2}(\xi)
\]

Eq. (25) becomes

\[
\int_0^t dX_2(\xi,t') J(t,t') = X_{e \text{ pres}}(\xi)(J(t,t_0)+\Phi(t,t_0)(g_2(\xi)-1))
\]

having the solution

\[
X_2(\xi,t) = X_{e \text{ pres}}(1+(1-R(t,t_0)/E)(g_2(\xi)-1))
\]

The solution of the problem takes so the final form

\[
X(\xi,t) = X_1(\xi,t) + X_2(\xi,t)
\]

The computation of the solution requires the relaxation function \( R(t,t_0) \) to be determined. This function can be derived directly by integration of Eq. (3) for \( \varepsilon = 1 \), thus obtaining the differential equation

\[
dR(t,t_0)/d\Phi + R(t,t_0) = 0
\]

with the condition

\[
R(t_0,t_0) = E
\]

so that it results

\[
R(t,t_0)/E = e^{-\Phi(t,t_0)}
\]

Otherwise, the following algebraic formulation can be employed

\[
1-R(t,t_0)/E = \Phi(t,t_0)(1+\chi(t,t_0)\Phi(t,t_0))
\]

where for \( \chi(t,t_0) \) the approximate value \( \chi = 0.8 \) can be assumed.

Introducing the functions
Eqs. (23), (28), take the final form
\[ X_{1}(ξ,t) = \Phi(ξ,t)/(1 + \chi(t,t_0)\Phi(t,t_0)) \]

\[ X_{2}(ξ,t) = 1 + (g_2(ξ) - 1)\Phi(t,t_0)/(1 + \chi(t,t_0)\Phi(t,t_0)) \]

Eqs. (23), (28), take the final form
\[ X_{1}(ξ,t) = X_{e}^{\text{post}}(ξ)k_{1}(ξ,t) \]

\[ X_{2}(ξ,t) = X_{e}^{\text{pres}}(ξ)k_{2}(ξ,t) \]

Observing that the functions \( g_{1}(ξ) \), \( g_{2}(ξ) \) are less than unit, and that the same happens for \( k_{1}(ξ,t) \), \( k_{2}(ξ,t) \), it can immediately be derived that for each of the components \( X_{i} \) of the vectors in Eqs. (36), (37), it results
\[ |X_{1i}(ξ,t)| \leq |X_{e}^{\text{post}}(ξ)| ; |X_{2i}(ξ,t)| \leq |X_{e}^{\text{pres}}(ξ)| \]

but, since for the analysis that does not take into account the construction phase, for the vector of the statically indeterminate solutions \( X_{e}^{*}(ξ) \) the following relation holds true
\[ X_{e}^{*(ξ)} = X_{e}^{\text{post}}(ξ) + X_{e}^{\text{pres}}(ξ) \]

It is thus immediate to note that
\[ |X_{i}(ξ,t)| = |X_{1i}(ξ,t)| + |X_{2i}(ξ,t)| \leq |X_{e}^{*}(ξ)| \]

The functions \( k_{1}, k_{2}, g_{1}, g_{2} \) are represented in Fig.5 for \( t=\infty \).

**Fig.5 – Functions \( k_{1}, k_{2}, g_{1}, g_{2} \)**

The components of the statically indeterminate actions are thus lower than those calculated by means of a conventional analysis in the elastic field, when the construction phases and the ageing of columns are neglected, so that the solution is found in the framework of the first theorem of linear viscoelasticity. As a consequence, the redistribution of the flexural redundant reactions caused by the column shortening, taking into account creep deformations and construction phases, is lower than the one evaluated by means of a conventional analysis. For this reason, for the safety verifications of the cross sections at the continuity supports of the slab beams, the most severe solution is the one where the columns are assumed to be infinitely rigid. In the same way, the solution derived from a conventional analysis in the elastic field is the most severe condition for the cross sections subjected to positive bending moments. In order to carry out the safety verifications of the cross sections of the beams, it is so sufficient and safe to refer to the two above mentioned limiting cases.

**3. Numerical analyses**

In order to determine the final displacements of columns at \( t=\infty \) and the actions that they cause in the peripheral beams, at first the viscoelastic properties of the material must be evaluated, according to
the CEB/FIP Model Code 90 /5/, assuming that for this model the ageing of concrete can be described by Eq. (8)
Referring to a mean diameter of columns $D_n = (120+65)/2 = 92.5$ cm, corresponding to which the notional thickness is:
$$h_0 = 2A/p = D_n/2 = 46.25$$ cm
with a relative humidity RH = 80%, characteristic strenght $f_{ck} = 45$MPa, and assuming $t_c = 10$ days as the time of loading, the following parameters are derived:

$$E_c = \beta_c (t_0) \cdot E_{c28} = \left(1 + \frac{0.25 (1 - \sqrt{25/\lambda_2})}{1} \right)^{1/2} \cdot 2.15 \cdot 10^4 \cdot \left(\frac{f_{ck} + 61}{10}\right)^{1/3} = 34460$$ MPa

$$\Phi_{RH} = \frac{1}{1 + \frac{1 - RH/100}{0.45 - 0.3}} = 1.26$$

$$\beta(f_{cm}) = \sqrt{f_{cm} + 0.13} = 2.30$$

$$\beta(t_{0c}) = \sqrt{t_{0c} + 0.13} = 0.59$$
from which

$$\Phi^- = 1.26 \times 2.3 \times 0.59 = 1.72$$
As for the rebar, its effect on the reduction of creep deformations, according to /9/, they can be taken into account by referring to the varied creep coefficient:

$$\Phi^* = \Phi^- (1 - \omega)/(1 + \omega \cdot \Phi^-)$$
being

$$\omega = 1/(1 + 1/(a_c \rho_x))$$
and assuming

$$a_c = E_c/E_c$$
$$\rho_x = A_x/A_c$$
with the above numerical values, taking into account that the mean geometric rebar ratio at the bottom of the columns is about 3.6%, it can be written:

$$\omega = \frac{1}{1 + 1/(a_c \rho_x)} = 0.1727$$
so that

$$\Phi^* = 1.72 \cdot (1 - 0.1727)/(1 + 0.1727 \cdot 0.8 \cdot 1.72) = 1.15$$
This value is the one to be used for the evaluation of the long term displacements. As for the value of $q/A$, representing the maximum normal stress at the bottom of the columns, where the rebar ratio reaches its maximum value $\rho_{yx} = 3.6\%$, the transformed area of the column $A^*$ becomes equal to 1.21 $A_c$. Referring to the peripheral beams of the slab, represented in Fig. 6, supported by columns 44, 46, 52, 53 and by the core, structural analyses, performed in the elastic field, provided the following stresses and maximum displacement, respectively expressed in MPa and cm, reported in Tab.1.

| Tab.1 – Values of stresses and displacements at selected columns from structural analyses |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Column 1-044                     | Column 1-046    | Column 1-052    | Column 1-053    | Core            |
| 8.14                            | 11.25           | 12.24           | 12.48           | 8.48            |
| $u_{01}$ : Column 1-044          | $u_{02}$ : Column 1-046 | $u_{03}$ : Column 1-052 | $u_{04}$ : Column 1-053 | $u_{0N}$ : Core |
| 3.60                            | 4.97            | 5.41            | 5.51            | 3.75            |

Using the values derived as described above, the curves represented in Fig.7-13 were plotted, reporting the displacements of the cross sections of the columns and the core at the different heights.
Fig. 6 – Position of the analyzed columns and core in the plan of Core1 (i.e. the ‘Tower’ building in the Palazzo Lombardia Complex in Milan)

Fig. 7 – Displacements of Column 1-044

Fig. 8 – Displacements of Column 1-046

Fig. 9 – Displacements of Column 1-052

Fig. 10 – Displacements of Column 1-053
The maximum displacements of the columns at the initial time are lower than 1.5 cm and are reached at half of the height of the columns. These displacements increase in time and their maximum values, which are reached in the cross section at a height of about 6/10 of the total column height, are lower than 3 cm. The creep of concrete thus causes a final maximum value of displacements which is about twice as high as the initial elastic one. The absolute increase of the displacements in time is about 1.5 cm, as for the relative displacements between the columns and the core, there are initial differences in the range of 0.5 cm, that increase in time up to a maximum value lower than 1 cm. An increase of the maximum differential displacements of about 0.5 cm can be observed because of the creep of concrete. The maximum increase of the vertical differential displacement between Columns 1-044 and 1-046 is of about 4 mm and gives way to a mean rigid rotation, measured along the height of the facades, equal to $\theta = 0.4/630 = 1/1575$ being 6.30 m the distance between the two columns. This rotation is compliant with the tolerances allowed by the designers of the facades, so that it can be concluded that the required performance level of the facades is not affected by creep phenomena in concrete.

Once the displacements in the columns are known, it is possible to directly evaluate the bending moments they induce in the peripheral beams. These moments were plotted, both for the conventional structural analysis and for the one taking into account the construction phases.

As above mentioned, the bending moments at the initial and at the final time computed by means of an analysis that takes into account the construction phases are lower than those computed by means of a conventional analysis. In particular, the total bending moment pattern is derived by superimposing a first pattern computed by assuming the supports i.e. the columns, to be infinitely rigid to a second pattern induced by the vertical displacements of the columns. The corresponding diagrams, both referring to an elastic behavior of the material and computed by means of the first theorem of linear viscoelasticity, without taking into account the construction phases, are reported in Fig. 14-15-16.
In Fig. 17-18 the diagrams corresponding to the analyses that take into account the construction phases are reported, and their superposition to the solution with rigid supports. In Fig. 19 the flexural moments caused by the displacements of the columns for the two situations are reported: one taking into account construction phases, the second neglecting the construction phases. In the same figure, the final results for the cases of rigid supports and displaced supports are reported, taking into account the construction phases and neglecting them. As previously theoretically derived, the diagrams derived from the analysis that takes into account the construction phases (green line) are intermediate between the ones from the analysis with rigid supports (red line) and higher than those from the elastic analysis on the homogeneous structure (blue line).

These results are practically coincident in the end bays, because the end columns have almost negligible differential displacements. The above derived results are quite meaningful, because they allow the cross section verifications of the slab, for flexure and for punching shear, to be reduced to
elastic verifications on a structure with rigid supports and on a structure with displaced supports. It can thus be concluded that, taking into account the construction phases, no worse scenarios will take place in time than those covered by the verifications made under the assumption of rigid supports or elastically deformable columns.

Fig. 20 – Flexural moments in the service life limit state under total loads, taking into account the construction phases

![Fig. 20 – Flexural moments in the service life limit state under total loads, taking into account the construction phases](image)

Fig. 21 – Flexural moment on the beam in the service life limit state

![Fig. 21 – Flexural moment on the beam in the service life limit state](image)

Some more remarks can be made as to the differential deformations induced by the shrinkage of concrete and the consequent vertical displacements taking place in the columns and in the core. Shrinkage induced deformations in a concrete cross section give way to a total deformation that, once the phenomena have completely developed, can be written as:

\[ \varepsilon_{CS} = \varepsilon_{CS0}(1 - \beta_s(t_f - t_i)) \]  

(30)

Being \( \varepsilon_{CS} \) the final shrinkage deformation. In the present case study, the effects of shrinkage must be taken into account starting from the time \( t_f - t_i \) which represents the end of the construction, corresponding to about one year after the first column was cast. Referring to the FIP/CEB Model, being the notional thickness of the core equal to 400 mm, in the two cases it can be derived:

\[ \varepsilon_{CS,c} = 0.000261 - 0.000064 = 0.000197 \] for the core

\[ \varepsilon_{CS,\text{c}} = 0.000261 - 0.000056 = 0.000205 \] for the columns

Also, since the deformation \( \varepsilon_{CS} \) is constant along the height, the vertical displacements of the core and the columns can be written as:

\[ u_{CS,c} = u_{CS,\text{c}} = \varepsilon_{CS,c} \xi \]  

(31)

\[ u_{CS,c} = u_{CS,\text{c}} = \varepsilon_{CS,\text{c}} \xi \]  

(32)

The two functions (31), (32) are plotted in Fig. 22. It can be observed that, at the final time, the differential vertical displacement between columns and beams due to shrinkage, at a height equal to 6/10 of the total column height, has an absolute value of about 22mm. No meaningful differential displacements take place between the core and the columns, so that the shrinkage induced vertical displacement appears to be practically uniform and does not worsen the stresses in the slab system.
As for the facades, shrinkage causes interstorey drifts of about $\Delta h = 0.22\ mm$, which are compliant with the good service life behaviour of the facades.

Fig. 22 – Shrinkage induced displacements

4. Conclusion

The analysis of the long term column shortening in tall buildings, carried assuming a viscoelastic rate of creep ageing behavior of concrete, allowed a simple and safe-side approximated approach to be formulated, for the evaluation of the stress distribution in the slab and for the definition of the upper and lower bounds of their values in time. The solution of the problem discussed in the present paper allows the generic rheological inhomogeneity of the material along the height to be taken into account, so that, besides the case of structures with reinforced concrete columns, for which rheological inhomogeneity is due to the different ages of casting of the columns themselves, other cases can also be studied with the same method such as that of concrete slabs interacting with steel columns and concrete cores, which induces a plan-wise rheological inhomogeneity in addition to the one along the height. The upper and lower bounds derived for the solution, assuming a rate of creep behavior of concrete, hold true also in presence of a generic creep law for concrete, so that it is possible to reliably approximate the solution of the problem in a generic situation. For this evaluation, only elastic computations are necessary, which can be carried out without taking into account the phases of construction. This represent an important advantage in the early design process, because it limits the need for complex finite element models to be created and makes them only necessary in later phases, to refine the initial simplified calculations and the conceptual design choices previously made.

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